Leibniz's *Monadology* and the Philosophical Foundations of Non-locality in Quantum Mechanics.

by

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Abstract:

One of the most troubling aspects of our understanding of modern physics, generally, and quantum mechanics, specifically, is the concept of "non-locality." Non-locality appears in an entire class of experiments, including the so-called "two-slit" experiment. In these, particles and "quanta" of light can be emitted and absorbed individually. Yet in the way these particles or quanta traverse the space and time between emission and absorption, they appear to behave not as point particles, but as though they were distributed throughout the entire spatial volume and temporal extent of the experiment. That the phenomenon of non-locality has recently been corroborated over macroscopic distances of the order of 10 kilometers makes these effects all the more remarkable.

In this lecture, I shall review the experiments and arguments that have led to an acceptance of non-locality in modern physics, and will suggest that the concept of space and time that this understanding implies is consistent with Leibniz's *Monadology*, wherein our ideas of space and time are fundamentally different from those given to us by our intuitions.

Outline of Lecture:

1. Leibniz's Monadology
   - Principle of sufficient reason
   - Leibniz-Clark Correspondence
   - Leibniz and Newton

2. Quantum Mechanics: An Eternal, Golden Braid
   - Einstein's paper on the photoelectric effect
   - Rutherford, Bohr, de Broglie, and Heisenberg: an eternal golden braid
   - The Heisenberg Uncertainty Principle
   - Einstein, Padolski, and Rosen's response to Heisenberg and Bohr

3. Bell's Theorem (or Bell's Inequality)
   - Bell's work at CERN
     - a synopsis of Bell's inequality via entangled particles
     - Bell's untimely death
   - Henry Pierce Stapp's paper on Bell's Theorem and it's implications
     - three reasonable demands: locality, causality, and individuality
     - must abandon one of the three
     - best option: abandon locality
   - Experimental foundation: Stern-Gerlach correlation experiments
and more recent experiments with quanta of light
Professor Carol Alley's indignation
   - What is the Speed of Quantum Information?" 18 km, 10,000 times speed of light.
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4. Like shadows on a Cave Wall: Leibniz's ideas of “space” as a kind of answer to the problem of non-locality

5. Concluding remarks
   - The coherence of mind—*The Emporer's New Mind*: Roger Penrose on the unity of cognition
   - The commonality of experience
   - The problem and promise of entanglement

1. Leibniz's *Monadology*

Principle of sufficient reason
Leibniz-Clark Correspondence
Leibniz and Newton

Leibniz's writings on the philosophical, mathematical, and natural sciences represent a coherent, if somewhat surprising whole. Nowhere is this more clearly illustrated than in the *Monadology,* the *Discourse of Metaphysics,* and the Leibniz-Clark correspondence.

Leibniz begins with the view of God as a maker, a being who makes the world the best possible.

Part and parcel with this view of the world is Leibniz's *Principle of Sufficient Reason*. It goes something like this: one monad can only be different from another because of its different character or qualities. I'll use a modern idea of a monad to illustrate this: an elementary particle like an electron. I hope my choice will become plausible a bit later when we start the discussion of quantum mechanics.

Is one electron the same as another? If so, if there is NO difference between "this electron" and "that electron", then they would be the same, since by the *Principle of Sufficient Reason* they cannot be distinguished. But my simply pointing to them is an indication of the differences. "This electron" IS different from that one, because it has an explicitly different representation that is indicated by my pointing at them. If I were to insist on a Cartesian representation of this difference, I can make a three-dimensional coordinate system with a particular origin and three orthogonal axes, labeled x, y, and z. Numbering these axes, I can locate "this" electron and distinguish it from "that" electron by the use of three numbers, *x1, y1,* and *z1,* and *x2, y2,* and *z2*. I can then say that I have a representation of each of these two electrons as different, given these two sets of three numbers. I can even represent their separation of this electron from that electron by a three-dimensional version of the Pythagorean theorem.

Leibniz makes this explicit several places in his works. For example, in the essay, "On Nature Itself (pp 164 – 165), he states in arguing against Descartes' reliance on geometry in physics. Given such an identity or similarity between objects,

> … not even an angel could find any difference between its states at different times, nor have any evidence for discerning whether the enclosed sphere is at rest or revolves, and what law of motion it follows....Even if those who have not penetrated these matters deeply enough may not have noticed this, it ought to be accepted as certain that such consequences are alien to the nature and order of things, and that nowhere are there things perfectly similar (which is among my [Leibniz's] new and important axioms). (Leibniz's essay, *On Nature Itself*, paragraph 13)

Of course, electrons have other properties as well: charge, mass, angular momentum (they seem to spin like tops), magnetic moment (they act like tiny bar-magnets), velocity, momentum, and kinetic energy, among other things. Each of these qualities or characteristics can also be represented by a series of numbers or "coordinate expression." I've always fancied that in a very formal sense an electron or any other elementary particle (had Leibniz known about them) could be represented as an aggregation of numbers (or coordinate expressions) related to another monad. This other monad could also be represented in a similar way. By the Principle of Sufficient Reason, some of these coordinate expressions are different from the coordinate expressions of all other monads.

The other thing to mention about monads is their unity. They are "simple." They do not have parts.

According to Leibniz, they represent a unity of different properties, much like a geometric point that is the nexus of many geometric lines. Leibniz states that:

> Everything is full in nature... And since everything is connected because of the plenitude of the world, and since each body acts on every other body, more or less, in proportion to its distance, and is itself affected by the other through reaction, it follows that each monad is a living mirror or a mirror endowed with internal action, which represents the universe from its own point of view and is as ordered as the universe itself. Leibniz, *Principles of Nature and Grace, Based on Reason,* paragraph 3.

Some even have the property of being "be-souled." So look around you. According to Leibniz, you are sitting among a reasonably large group of monads, each of which is capable of noticing you and regarding you as separate, individual "beings."

There is one final thing about monads (among their many interesting properties) that bears on our discussion of quantum mechanics. As Leibniz says, tt another point, in the *Monadology:*

> The monads have no windows through which something can enter or leave. (*Monadology,* paragraph 7)

Monads have no "windows." Yet each monad is a representation to a greater or lesser extent of everything else in the Universe because it is linked to all other monads by means of its relation to God. That is, each monad is a reflection of the entire Universe precisely because it is in some way a projection of a part of God. The debt Leibniz owes to Plato's *Republic* for this concept (note that I did not say image) is nowhere directly acknowledged by Leibniz, but it is manifest. The one quarrel Leibniz would have with my associating him with the image of the Cave in the Socratic dialog is simply that it is an image as opposed to something that dwells in the understanding. For Leibniz's God is, at least to my thinking, a Mathematician, and He, like Dedekind, holds that mathematics has no need of geometry.

In this conception, then, there is a profound similarity between all of our connections with one another and with the physical, social, and moral world.

It seems clear, therefore, that Leibniz does not think that space has an actual existence. As he states explicitly,

> As for my own opinion, I have said more than once that I hold space to be something relative, as time is, that I hold it to be an order of coexistences, as time is an order of successions. (*Letters to Clark*, Leibniz's Third Paper, Paragraph 4)

This is radically at odds with Newton's *Principia*, wherein Newton seems to deduce the existence of absolute space from the existence of absolute (i.e, accelerated) motion. For Newton, space is the "sensorium of God."

Let us ponder this for a moment. For Newton, space has an existence. We can look out into the space before us and hold it in our minds as *something*, even though we can (as Kant does) in our imaginations remove all of its contents from the space that holds it. What is left over is space, be it a cubic centimeter in front of us or volume 100,000 parsecs on a side.

When Leibniz sees this emptiness, he views it as an actual *metaphysical* void, something that not even God can relate to. As such, it is an abomination. Leibniz cannot accept a thing that God cannot act upon, and the idea of an actual void is such a thing. Since God must be able to act on all creation, a genuine *metaphysical* void cannot exist. This is one of the reasons why the Leibniz-Clark correspondence (Clark was taking Newton's part) makes little headway to change the authors' minds. The grounds of the conversation are radically different.

It is a worthy anecdote to relate that Leibniz and Newton never acknowledged the other's invention of the differential and integral calculus. And it is helpful to note that Newton's development of the calculus relies on geometrical constructions, while Leibniz's relies on an evolution of Descartes' algebra. Is it true that Leibniz uses sketches of curves and lines for his derivations, in part because we are visual creatures, but Leibniz's derivations do seem to be less reliant on images of extension.

Thus, for Leibniz, extension has no actual existence. What we interpret as extension, as space, is a representation given to us by God. It is very likely that the same is true for time in Leibniz's metaphysics. This separation is like a three-dimensional Pythagorean theorem whose terms are given to us. What we interpret as a spatial extension is a coordinate interval that we call space, just as temporal separation is a coordinate expression that we call time. What separates us, what we interpret as distance, is just a shadow on a Cave wall caused by our origin within a common light. What separates us from the amber light of ages past is an equivalent coordinate expression whose regularity is provided by God.

I cannot resist at this point recalling for you the yarn in the *Odyssey* when the hero is among the *Phaeacians,* and Homer brings us back from the story Odysseus is telling into Alkinoos and Arete's palace hall with it's feast and polished stone floors and torchlight. The momentum of that telescoping does not stop there, but places us back firmly into the present where we realize that we are reading words two thousand years old about a story that is a thousand years distant even from that remote past. Like Leibniz's God, Homer has linked us to the ages, and three millennia are as nought.

One other element of Leibniz's philosophy will prove useful later: Leibniz directly addresses the problem of a Deity that weaves out our destiny to construct the best of all possible worlds. This Deity knows everything we are capable of doing, knows all of our potentialities, and further, knows all of our past.

> And since every present state of a simple substance is a natural consequence of its preceding state, the present is pregnant with the future (*Monadology*, paragraph 22).

Thus, the “Demon” in Laplace's *Essay on a Theory of Probability* takes its tack from Leibniz. Laplace says explicitly:

> We ought then to regard the present state of the universe as the effect of its anterior state and as the cause of the one which is to follow. Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it -- an intelligence sufficiently vast to submit these data to analysis-it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes. (Laplace, *A Philosophical Essay on Probabilities*, ch II)

Leibniz seems to recognize the determinism of such a God, but sidesteps the troublesome argument of the lack of free will by claiming that God knows all possible predicates of our being, and so chooses the path which we would follow anyway!

I regard the foregoing comments about Leibniz's *Monadology* as a preamble to our discussion of the problem of non-locality in quantum mechanics, especially as the concept of non-locality has been articulated by interpretations of the work of John Bell, an elementary particle theorist who worked at CERN before his untimely death in the Fall of 1990. But first, I shall try to provide some background on the landscape in which Bell developed his certifiably famous theorem.

2. An Eternal, Golden Braid: Quantum Mechanics in Rutherford, Bohr, de Broglie, Heisenberg, and Einstein

Einstein's paper on the photoelectric effect
Rutherford, Bohr, de Broglie, and Heisenberg: an eternal golden braid
The Heisenberg Uncertainty Principle
Einstein, Padolski, and Rosen's response to Heisenberg and Bohr

It is surprising at first gloss that of the four papers Einstein published in 1905, the one for which he was awarded the Nobel Prize in Physics was not one of the following:

The one on Special Relativity, entitled "On the Electrodynamics of Moving Bodies". *Annalen der Physik* 17 (1905): 891-921;

nor the one entitled

"Does the Inertia of a Body Depend upon its Energy Content?" (the famous $E=mc^2$ paper), in *Annalen der Physik* 18 (1905);

nor the one on Brownian motion, entitled

"On the Movement of Small Particles Suspended in Stationary Liquids Required by the Molecular-Kinetic Theory of Heat." in *Annalen der Physik* 17 (1905): 549-560.

As an aside, it is worthy of note that this is the 100th anniversary of the publication of the 1915 paper on General Relativity, and the 150th anniversary of Maxwell's publication of his theory of light as electromagnetic waves.

The actual phrasing from the Nobel Prize Committee was "*for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect."* The so-called "photoelectric effect" paper has a curious title: "Concerning an Heuristic Point of View Toward the Emission and Transformation of Light." It was also published in *Annalen der Physik* 17 (1905): 132-148. It was one which marked the beginnings of what is now called Quantum Mechanics.

In the paper, Einstein characterizes the wave theory of light in the following manner:

> ...the energy of a beam of light from a point source (according to Maxwell's theory of light or, more generally, according to any wave theory) is continuously spread over an ever increasing volume.

In the next paragraph, Einstein notes that

> The wave theory of light, which operates with continuous spatial functions, has worked well in the representation of purely optical phenomena and will probably never be replaced by any other theory.

But in the next paragraph, he states that

> It seems to me that the observations associated with blackbody radiation, fluorescence, the production of cathode rays by ultraviolet light, and other related phenomena connected with the emission and transformation of light … are more readily understood if one assumes that the energy of light is discontinuously distributed in space. In accordance with the assumption to be considered here, the energy of a light ray spreading out from a point source is not continuously distributed over increasing space but consists of a finite number of energy quanta which are localized at points in space, which move without dividing, and which can only be produced and absorbed as complete units.

On the one hand, Einstein allows for a “wave theory” like Maxwell's waves in a luminiferous aether wherein the light it transmitted, reflected, and refracted. He “heuristically” considers light to be a particle during light's emission from and absorption into material bodies. It is perhaps ironic that Einstein was never able to reconcile his conception of the dual nature of light with the equivalent, dual character of particles as both material bodies and waves, a solution posed by de Broglie to provide an explanation of Bohr's model for the energy levels of the hydrogen atom.

Of course, this entire “braid” began with efforts to apply models from classical physics that explain everything from cannonballs to asteroids to planets to the very small structures within matter such as atoms and elementary particles via Galileo, Thomson, Millikan, and Rutherford.

By way of a truncated outline of the argument, Bohr used the existence of hydrogen spectral lines and the contemporary work by Planck to explain so-called Blackbody radiation. Planck made the hypothesis that discrete oscillators in matter had only certain fundamental modes with which they could vibrate. He asserted that these oscillators were in equilibrium with the thermal radiation from matter with a particular temperature, and thus explained blackbody radiation. Bohr wondered what the “Planck oscillators” could be, since the classical picture of an orbiting charge holds that it should radiate continuously. He hypothesized that his atom settled into quasi-stationary states and emitted and absorbed radiation during transitions from one energy level to another.

It is likely that everyone in the audience is familiar with Bohr's model from high school science classes and many popular lectures and books on the subject of science. You Seniors are in the process of completing this sequence of papers.

In fact, the Bohr model has become a commonplace picture of the atom. But such familiarity hides the utter strangeness of the concept. The atom is stable for a while, and then is excited or de-excited by the absorption or emission of light at a specific frequency. These energy levels are Bohr's answer to why the spectra of light from certain gases contains only certain frequencies. If you sprinkle salt onto the logs in your fireplace, the resultant light is a brilliant yellow. That yellow light contains only certain frequencies, frequencies that are as much an indication of the presence of the sodium in salt as your finger prints are of you as an individual person. We know the constitution of stars precisely because of this line-spectrum identification of elements, stars that can be hundreds or thousands of light years distant.

The strangeness of the idea of the Bohr atom bothered de Broglie, who reasoned by a kind of symmetry derived from Einstein's photoelectric effect paper (wherein light can have a particulate nature, as well as a wave-like nature) that particles could perhaps have both a discrete nature and also a wave-like nature. In an immensely clever argument (he won the Nobel Prize for it), de Broglie argued that one can calculate the "wavelength" of a particle by assigning it a specific momentum, which implies that it has an energy. That energy can be used to calculate a characteristic wavelength, $E = h\nu = hc/\lambda$. It is a stunning triumph for so simple an argument that the wavelengths thus calculated for an electron in the Bohr orbits for hydrogen is exactly the circumference of the quasi-stationary orbits for electrons in the hydrogen atom. So the electrons are not exactly particles when they are inside the atom. They also have wave-like qualities.

Schroedinger was a young assistant professor when de Broglie published his astonishing idea. I have it on good authority that Schroedinger was assigned the task of giving the journal club lecture at his university the next week. It's a bit like these Friday night lectures, but less formal and typically they are on a week-day afternoon. The assignment was something like, "Take a look at de Broglie's paper and give us a synopsis of it at the journal club next Tuesday."

Schroedinger had a ski trip planned for that weekend (Friday through Sunday, apparently). Being the persistent soul that he was, he took a copy of de Broglie's paper and a book on solutions to differential equations in various coordinate systems (rectilinear, cylindrical, and spherical) with him on the ski trip. The short version of the story is that he didn't get much skiing done, but he came back well on the way of inventing wave mechanics, an explanation for the energy levels of atoms as kind of standing waves in space. His "eureka" moment came when he said to his bewildered ski companions, "I have just fit the energy levels of the hydrogen atom in a way you would not believe!" The standing waves were similar to the three-dimensional oscillations of sound waves in a concert hall. But standing waves of what?

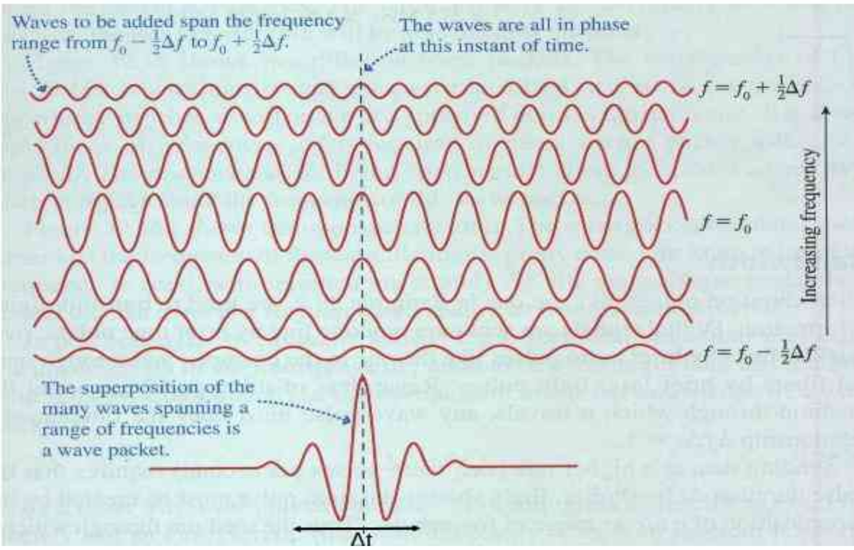
I believe Schroedinger originally thought of the standing waves as waves of charge density. The electron has wave-like qualities à la de Broglie, and it has charge, so it would make sense as an extension of de Broglie's hypothesis. But electrons have discrete charges when they are measured by Millikan in his famous oil-drop experiment. How come we never see fractional charges?

Schroedinger's description of electrons (or any elementary particle, for that matter) was that they are aggregations of waves that reinforce in a certain region and cancel out everywhere else. This makes sense in explaining the energy levels of a hydrogen atom, but causes other conceptual problems.

First slide (the Cat – on title page)

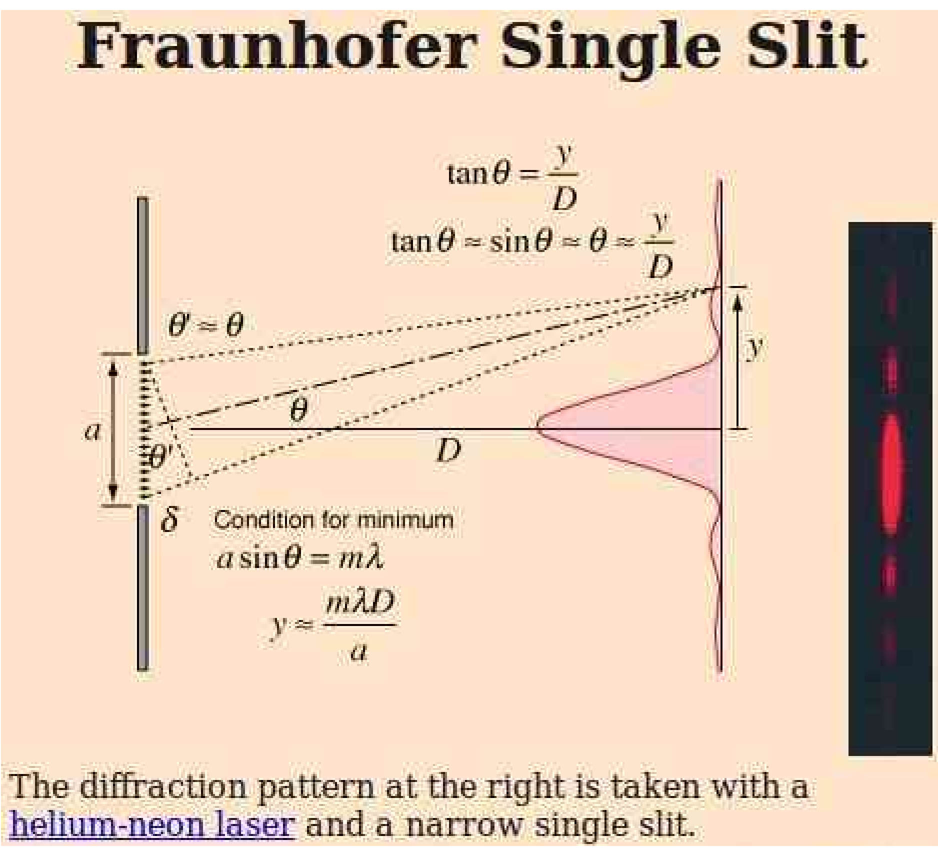
Schroedinger's description of a particle as an aggregation of waves of some sort caused Heisenberg to analyze the behavior of such particles when we try to measure them. If we try to localize the particle as we do in the act of measurement, we confine it to a narrower region in space. That means we add up more and more waves. Each wave has a slightly different speed. Schroedinger needed these different speeds for different wavelengths in order to get the "wave-packet" to behave like a particle. But that means that the momentum of the particle becomes less certain over time, since, in order to localize the particle, we need to add more wavelengths, and adding more wavelengths means the velocity (and therefore the momentum) become more uncertain.

Slide 2: Slide of wave addition to produce wave-packet goes here



There is actually a calculable limit to the uncertainty in the momentum times the uncertainty in the position of a particle. It is greater than or equal to Planck's constant. This is of course the Heisenberg Uncertainty Relation. It says that there is a fundamental, and not simply an experimental, limit to our knowledge of the location of a particle and its momentum.

Slide 3: Slide of single slit diffraction and its relation to the Heisenberg Uncertainty Principle



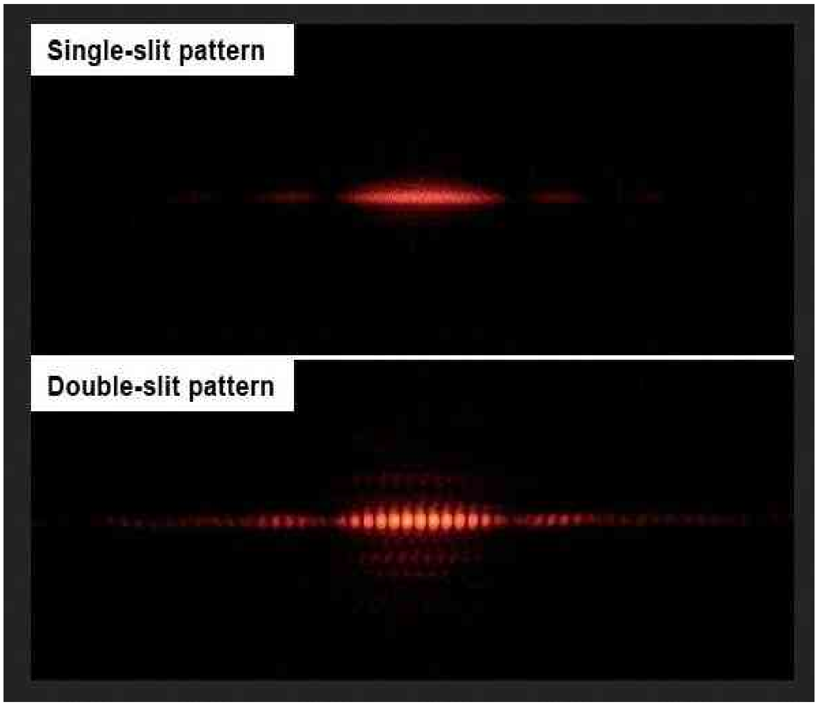
A particularly helpful illustration of the Heisenberg derivation (and one that will be useful to us later in this lecture can be had by looking at single-slit diffraction of a plane wave. The wave can be a wave of light, an elementary particle like an electron, or even a water wave. If it originates from a far-distant source, the wave is essentially a series of parallel troughs and peaks with its propagation direction perpendicular to those troughs and peaks. When we allow it to approach a screen so that the peaks and troughs (as seen from above) are parallel to the screen, we can watch the interaction of the barrier with the oncoming waves. If there is an opening in the barrier that is of the same order as the wavelength of the waves, a fraction of the waves can pass the barrier. When this happens, a part of the wave front gets through the barrier, but for some fraction of the waves, the direction of the waves is changed because of the wavesfront's interference with itself. This interference produces a dispersion of the wave front that gives its velocity a vertical component. It is important to note what has happened here. We have limited the wavefront in the vertical direction to a Δx that is essentially the width of the slit. It has produced a dispersion in the velocity of the wave in the vertical direction, a Δv.

In Schroedinger's terms, this dispersion in the velocity of the wave in the vertical direction (that is, in the same direction as the opening of the slit) is an uncertainty in the velocity. If we consider the wave as representing the motion of a particle, then the localization of the particle within a delta x produces an uncertainty in the momentum of the particle of order delta p. This illustration is not entirely fanciful. In fact, Heisenberg uses it as one of his derivations of the Heisenberg Uncertainty relation. Furthermore, the smaller the slit, that is, the smaller the uncertainty in position, the greater the uncertainty in the momentum.

This has led to no end of problems in interpretation. One example of this is the fact that elementary particles (be they electrons, protons, or photons), when emitted from a source and directed toward a screen or grid whose spacings are the same size as the wavelengths of the elementary particles, will show a diffraction pattern on a screen downstream from slits. For the sake of clarity, we will consider only photons, although the discussion could as well apply to any elementary particle, including neutrons, protons, electrons, etc.

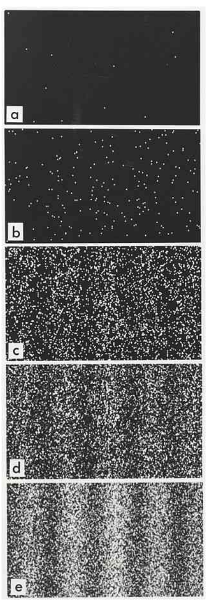
Let a stream of photons set forth across the chaotic gulf toward a screen. Imagine this as like a scene from Milton's *Paradise Lost* as Satan launches himself across the chasm between hell and paradise. These photons are transmitted and diffracted as though they are electromagnetic waves. When they reach two slits in the screen, the waves interfere with one another so that there is a very specific pattern of light and dark lines on the screen downstream from the slits called a “two-slit” pattern.

Slide 4: Slide on single-slit vs. double-slit pattern here



Suppose we turn down the intensity of the light.   Let us make the light exceedingly dim, so that when we look at the screen or detector, we find only one cell on the screen illuminated or exposed (you remember photographic film, I trust?) at a time.

Slide 5: Low Light/Flux level slide goes here



What happens next is remarkable. This figure shows the buildup over time of electrons in a two slit experiment at very low flux levels. We see one quantum at a time arriving .   As we watch, the diffraction pattern begins to develop. We see the characteristic two-slit pattern. But we have allowed only one quantum (in this case, electrons) to be emitted at a time. How can we possibly get a two-slit pattern. Such an experimental apparatus exists. The results from it behave exactly as I have said.

Apparently, the individual light or particle quantum goes through both slits at once. It is spread out over the entire space of the experimental screen (or more properly, the experimental volume) and then excites only one element of the detector. If this seems quixotic to you, it is. It is known as “the problem of measurement” in the vernacular of Shady Bend. The wave function (remember all those waves adding up to produce the wave packet) is spread out even for a single particle or quantum of light. The moment before it hits the detector screen, it is everywhere on the screen. At the next instant, it collapses into a single point. This is known as the “collapse of the wave function.” The collapse is apparently instantaneous. If these are material particles or quanta of light, they sort themselves into a single area on the screen instantaneously.

There were many objections to this explanation, not the least of which was that it violates causality. The wave-packet description of the two-slit experiment requires that the waves instantly collapse to a single point, after having, a moment in time before, occupied the whole of the experiment.

Bohr and Heisenberg made noble efforts to resolve this apparent contradiction by supposing that the wave function description of elementary particles was merely a calculation of likelihood or probability. Since probability is only a likelihood, the collapse of the wave function is merely the result of a measurement. And like any measurement, once it occurs, the answer is always, “Yes. That's what happened!”

Einstein would have none of it. His famous quote, “God does not play dice!”, about the so-called Copenhagen Interpretation of Quantum Mechanics was an indication of his objection to the probability interpretation of the Psi-function. In his view, there was an underlying causal relation between the elements of the experiments and their outcomes that was not represented by quantum mechanics (QM). Yet QM is a remarkably successful theoretical method.

In a paper in response to the probability interpretation of QM, Einstein, Podolski and Rosen (EPR) tried to show that the uncertainty relation developed by Heisenberg was flawed, and that some variations of the single or two slit experiment would give an inroad into figuring out precisely what the momentum and position of the particle would be. One of the thought experiments proposed to measure the momentum transferred to the screen by the impact of the particle, This (by conservation of momentum) would allow the particle momentum to be measured exactly, while the position would be localized to the region within the slit. But when one took into account the uncertainty in the position of the screen, the Heisenberg Uncertainty limit returned.

A variant of one of the thought experiments used two particles that interacted prior to the slit, and then had one transfer its momentum to one screen while another's position was determined independently. Again, by conservation of momentum the second particle's momentum and its position were to be determined beyond the Heisenberg limit. Each response to EPR by Heisenberg and Bohr led EPR to a further amplifications of the experimental apparatus. While the correspondence in the scientific literature led many to accept the Copenhagen Interpretation and the Heisenberg Uncertainty limit, Einstein was never able to believe the probabilistic nature of Bohr and Heisenberg's interpretation.

Yet the alternative to a probabilistic interpretation was an instantaneous collapse of a physical wave function. This instantaneous collapse would clearly exceed the speed of light, and thus render it difficult to accept, since the limiting speed of the transfer of information in Special Relativity is the velocity of light. This is one of the fundamental hypotheses of Special Relativity.

This led John Bell to a further analysis of the two slit experiment, and the theoretical development of Bell's Theorem (or Bell's Inequality), which has allowed many experimental test of locality, causality, and the predictions of quantum mechanics. It appears to contradict Einstein's hopes for a “hidden variable” theory, wherein true causality would be returned to the world. Apparently, this is not to be realized.

3. Bell's Theorem (or Bell's Inequality)

Bell's work at CERN

a synopsis of Bell's inequality via entangled particles

Bell's untimely death

Henry Pierce Stapp's paper on Bell's Theorem and it's implications

three reasonable demands: locality, causality, and individuality

must abandon one of the three

best option: abandon locality

Experimental foundation: Stern-Gerlach correlation experiments

and more recent experiments with quanta of light

Professor Carol Alley's indignation

But how does this happen? Bell's theorem is essentially a test of whether or not two particles, once they interact, can be separated enough so that their states do not influence one another. Remarkably, it is posed in such a way that it can be implemented as an experimental test.

Schroedinger called this phenomenon, wherein the wave function of two particles becomes joined by their interaction, an “entanglement” of the wave functions of the particles. And you recall that all particles have a wave function description that guides or governs their behavior.

This hypothesis bears on EPR's paper. To reiterate, if two particles interact, then the momentum of one could be determined by inference due to measuring the momentum of the other, since the momentum of the pair has to be conserved. At the same time, the position of the first particle, for which we inferred the momentum, could be accurately measured for its position as long as the pair were sufficiently far apart. Thus, the momentum and position of a particle could be measured at a precision which violated the Heisenberg uncertainty limit. At this point, EPR could claim that the Heisenberg Uncertainty relation was merely a practical limit, and that there was some underlying, governing relation which we simply needed to find, some sort of “hidden variable” that really determined the evolution of the system.

J.S. Bell was sympathetic to EPR's view. His theorem (called variously Bell's Theorem or Bell's Inequality) was an attempt to establish whether or not EPR's hypothesis could be tested experimentally. The experimental setup is remarkably simple, but not trivial. Two particles would be allowed to interact, to become “entangled,” and then would separate and go off in opposite directions. After a time, the particles would each be measured to determine their properties. As with the EPR paper, the hypothesis that their states could no longer interact would produce one result, whereas the hypothesis that their states were still entangled when they were measured would produce another result.

The next figure shows the results of one of the experimental tests of Bell's Theorem, in this case the orientation of the polarization of photons measured by two separated systems. The red (straight) line shows the limit of a “local, realistic” hypothesis, that is, that the results are uncorrelated. Any experimental result below the diagonal red line indicates a correlation (that is, an entanglement) between distant particles and their experimental apparatuses. Perhaps most important, the results predicted by QM show a very close agreement with the data!

In some later experimental tests, groups have tried to estimate the speed of the transmission of the correlations by changing slightly the timing of the setting of the measuring apparatuses. In a ground-breaking paper entitled “What is the Speed of Quantum Information,” the result of a measurement conducted at CERN is that the correlations happen at a velocity at least 10,000 times the speed of light over a distance of 18 kilometers. I say “at least” because the electronics of the experimental setup could not measure a faster correlation. So for all intents and purposes, this speed is a lower limit. The correlations occur effectively instantaneously.

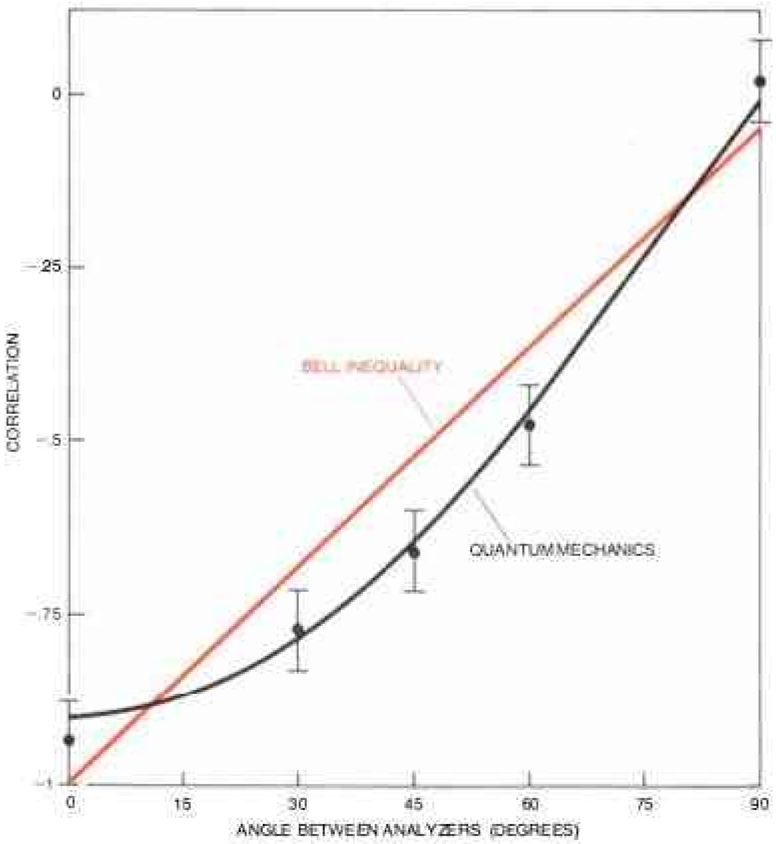
What are we to make of such results? Henry Pierce Stapp's paper, entitled “The S-Matrix Interpretation of Quantum Theory”, provides a highly recommended discussion of Bell's Theorem, despite the imposing title. But by way of a friendly warning, it's best to read Section X, Ontological Problems, and Appendix B, World View, first to get a bit of orientation.

To give you some idea of Stapp's take on Bell's Theorem, I quote from his paper at a point just after he shows a concise proof of that theorem.

> A conclusion that can be drawn from this theorem is that the demands of causality, locality, and individuality cannot be simultaneously maintained in the description of nature. Causality demands contingent predictions; locality demands local causes of localized results; individuality demands specification of individual results, not merely their probabilities.

For a more readable proof of the theorem, Nick Herbert's article, “Quantum Reality” and his account at http://quantumtantra.com/bell2.html, and in N. Herbert, Am Jour Phys **43**, 315 (1975) and in N. Herbert, New Scientist **111**, 41 (1986).

Slide 6: Bell's Theorem results.



RESULTS OF AN EXPERIMENTAL TEST of the Bell inequality show that it is clearly violated. The experiment is the one that employed pairs of protons in the singlet state, which was carried out by M. Lamehi-Rachti and W. Mittig of the Saclay Nuclear Research Center in France. The negative correlation between the values of different spin components is given as a function of the angle between the settings of the two analyzers. A correlation of −1 would indicate that the components invariably had opposite values. The Bell inequality states that the correlation at any angle must be on or above the colored line. The observed correlations at 30, 45 and 60 degrees are below the line. The results not only violate the Bell inequality but also are in good agreement with the predictions of quantum mechanics, which fact adds to their credibility. The violation of the Bell inequality implies that at least one of the three premises of local realistic theories must be false; Einstein separability is considered the most plausible candidate.

As Stapp puts it:

> I can see only three ways out of the problem posed by Bell's theorem.
> 1. The first is to accept, with Everett, the idea that human observers are cognizant only of individual branches of the full reality of the world: The full physical world would contain a superposition of a myriad of interconnected physical worlds of the kind we know. An individual observer would be personally aware of only one response of a macroscopic measuring device, but a full account of reality would include all the other possible outcomes on an equal footing, though perhaps with unequal “weights.”

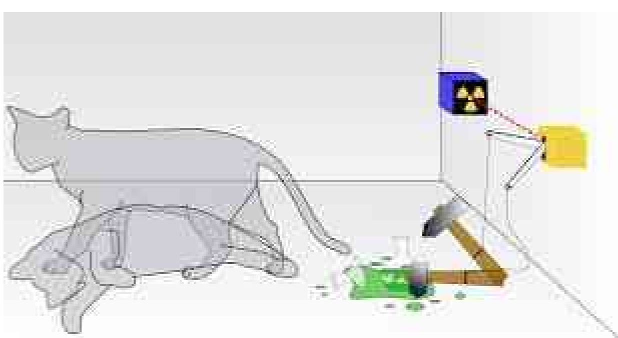
2. The second way out is to accept that nature is basically highly nonlocal, in the sense that correlations exist that violently contradict – even at the macroscopic level – the usual ideas of the space-time propagation of information. The intuitive idea of the physical distinctness of physically well-separated macroscopic objects then becomes open to question. And the intuitive idea of space itself is placed in jeopardy. For space is intimately connected to the space-time relationships that are naturally expressed in terms of it. If there are, between far-apart microscopic events, large instantaneous connections that do not respect spatial separation, then the significance of space would seem to arise only from the statistical relationships that do respect it.

3. The third way out is to deny that measurements that "could have been performed, but were not," would have had definite results if they had been performed. This way out seems, at first, to be closest to the spirit of the Copenhagen interpretation. However, it seems to contradict the idea of indeterminism, which is also an important element of the spirit of the Copenhagen interpretation. (Henry Pierce Stapp, 1971, Physics Review D, Vol. 3, no. 6, pp 1303-1320).

Some comments are clearly in order here. The third option Stapp articulates bears remarkable similarities to Laplace's Demon or Leibniz's God as architects of the best of all possible worlds. In that instantiation of reality, what we choose is exactly what we will. But what we will as a predicate of our being is completely known by the Deity and determined by it.

The first option is known as Stapp's "many-worlds" interpretation. That option is often mentioned in the same breath as Schroedinger's Cat.

Slide 7: Picture of Schroedinger's Cat here:



In that interpretation, as Stapp says, the cat is both alive and dead in the multiply unfolding universe of outcomes. Each point where the quantum hits the screen represents a starting point for a separate future.

As an interesting aside, we have some hopes of conducting Bell's Theorem type experiments here at St. John's in a room in the basement appropriately called the Quantum Lab. But of course, no cats will be allowed therein.

Most people find the second option, non-locality, most "appealing," if that is the right phrase.

In the case of the first experimental measurements, conducted with two low-energy neutrons colliding; then recoiling down separate arms of a vacuum line; and finally having their angular momenta determined by a Stern-Gerlach apparatus (I will spare you the details), there were (some thirty or forty years ago) five measurements, four of which agreed with Bell's inequality. Since then, all of the experimental tests of Bell's theorem have confirmed it.

To emphasize how surprising this has been, I recall a conversation I had with Professor Carol Alley at the University of Maryland when I was a graduate student there. He is a famous experimental physicist, one who used a laser to measure the distance to the Moon from a site near Goddard Space Flight Center during one of the Apollo Lunar Landing missions. As we talked about Bell's theorem, and it's apparent experimental corroboration, standing in the hallway in the Physics Building at the University of Maryland, he was clearly quite perplexed that there was any corroboration of the inequality. As we spoke, his voice was getting louder and louder. Finally, I said to him, “Professor Alley, you realize that you are shouting at me?” He laughed and said, “Well, it's certainly not you that I'm shouting at, Jim. It's the idea of this result!”

Left with the options Stapp articulated, which would you abandon: causality, locality, or individuality. You cannot have all three! Most people, faced with these options, give up locality.

4. Like shadows on a Cave Wall: Leibniz's ideas of “space” as a kind of answer to the problem of non-locality

It is time to recall one of the things I am attempting in this lecture: to use Leibniz's conceptions of space and time in the *Monadology* as a metaphysical foundation for the idea of no-locality in quantum mechanics.

Let us reiterate the properties of monads. Monads are singular. That is, they have many properties, but no parts. They have no windows. All their impressions and reflections of the Cosmos come through their reflection and articulation of the Deity, which they represent in a small part.

Finally, it is likely, based on the experimental results of Bell's Theorem, that our intuitions of space and time are far removed from the way the Universe actually is.

5. Concluding remarks

The coherence of mind - *The Emporer's New Mind*: Roger Penrose on the unity of cognition
The commonality of experience
The problem and promise of entanglement

I conclude this lecture with two principal points and some speculations.

First, it was many years ago that Roger Penrose in a book called *The Emporer's New Mind*, tried to explain the *coherence of mind* by the physical effects of non-locality on a relatively small scale – the electrochemical and quantum mechanical processes in the human brain (cats, also, most likely, since Penrose is fond of cats). This coherence would require entanglement of the prior physical states of these electrochemical wave fronts, but this does not seem terribly surprising.

Second, entanglement does not depend simply or perhaps even necessarily on proximity. At a fairly formal level, entanglement depends on interaction. The entanglement of cognitive processes with the experiential world might be sufficient to explain the *commonality of experience*, a term which I coin here in this essay, especially given that the correlations persist over manifestly macroscopic differences.

This bears, quite generally, on our ideas of culture, also. As an example, think of how easy or difficult it can be to change ones entire conception of the world via a single conversation. I thank Mr. William Braithwaite for the suggestion.

The concept of non-locality thus articulated can extend far beyond the possibility of common experience to the possibility of kindredness with our common weal. We might not, actually, be separate spheres, hoping to connect, hoping to touch and know the World. Like shadows on a Cave Wall, both we as individuals and the rest of the sensible world could actually be sprung from a common light.

Finally, and this is a bit more speculative, but hardly original, the entire evolution of the history of the Cosmos has involved some pretty heavy entanglement. We now call it the Big Bang.

This brings us to a further point regarding Leibniz's Deity. God might not have simply said, “Let there be Light.” God might have actually been that light.

Thank you.

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