THE ASTROPHYSICAL JOURNAL, 316:227–230, 1987 May 1

THE OBSERVATIONAL APPEARANCE OF PROTOSTELLAR ACCRETION DISKS

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Received 1986 April 21; accepted 1986 October 14

ABSTRACT

The observational appearance of a protostellar accretion disk has been calculated analytically by assuming a thin, thermally emitting disk with an unspecified power-law temperature gradient. The disk is assumed to have both a maximum and a minimum radius. The analytical spectrum is compared with data from the ρ Oph dark cloud taken by Lada and Wilking. Some of the spectra of sources in the ρ Oph cloud are shown to be consistent with the emission expected from a classic Shakura-Sunyaev disk when such a model is applied to the problem of protostellar formation (Lin).

Subject headings: stars: accretion — stars: formation

I. INTRODUCTION

Observations suggest that disks of accreting material are likely to occur in numerous astrophysical settings. These accretion disks have been suggested as being important in the dynamics of cataclysmic variable systems detected at optical and ultraviolet wavelengths (see, e.g., Smak 1985 for a review), in the origin of bipolar flows detected in the infrared (Welch *et al.* 1985), and in X-ray–emitting binary stars (Verbunt 1982). Accretion disks are also thought to play a direct role in the formation of stars and stellar systems (see, e.g., Lin 1981; Gehrz, Black, and Solomon 1984). Stahler, Shu, and Taam (1980) have both reviewed and extended the discussion of the models of spherical accretion as a means of forming stars. The precise role of accretion disks in the formation of planets is still debated (see, e.g., Cameron 1978; Safronov 1972; Hayashi, Nakazawa, and Adachi 1977; Wetherill 1978; Boss 1985). Wynn-Williams (1982) discusses the current, disappointing observational status for the detection of luminous ($>10^3\ L_\odot$) protostellar accretion disks in molecular clouds.

The structure of protostellar disks is extremely important when considering the question of the formation of planetary systems. Lewis (1974) has pointed out that the radial segregation of material in the solar system implies that the primordial solar nebula had a temperature gradient with a power-law index of approximately -1.1. To model protostellar disks, Lin (1981) and Lin and Papaloizou (1980) have assumed a plausible value of -0.75 for the power law of the effective disk temperature gradient corresponding to a midplane gradient of -1.5), using a modification of the Shakura-Sunyaev (1973) viscous disk model. Alternatively, Shakura and Sunyaev and others consider viscous accretion disk models wherein the variation of effective disk temperature with radius can be characterized by the midplane temperature gradient which follows a -0.75 power law for portions of the disk that radiate in the infrared. Since the -0.75 value of the power-law index of the disk temperature gradient is derived from very general and compelling considerations (see, e.g., Verbunt 1982 for a review), it becomes interesting to ask whether more direct observational tests of disk temperature gradients are available. Recently data from the *IRAS* satellite (Beichman *et al.* 1984) and ground-based observatories (see e.g., Lada and Wilking 1984; Grasdalen, Strom, and Strom 1973; Vrba *et al.* 1975; Elias 1978) have been reported for observations of infrared sources in dense molecular clouds. As these authors point out, such observations can, in principle, represent detections of protostellar systems.

In this paper, we calculate analytically the spectrum of emitted radiation from a disk of accreting material. We assume an optically thick, finite disk which is truncated at its inner edge by the protostar which has not yet begun nuclear burning. In view of the possible uncertainty of the temperature gradient in the disk of accreting material, we leave unspecified the power-law index of the disk temperature gradient which has the form $T = T_0 r^{-p}$, where T_0 is the disk temperature at the radius of the protostar, R_0, and $r = R/R_0$, where R is the distance from the center of the protostellar nebula. Within a finite range of frequencies determined by the central temperature of the accretion disk and the maximum disk radius, the emitted spectrum has the form $S_\nu \propto \nu^\alpha$, where $\alpha = [3 - (2/p)]$. Finally, we compare these results with the infrared data taken from observations of the ρ Oph molecular cloud (Lada and Wilking 1984). These data are chosen because they represent low-luminosity, low-mass systems. As Wynn-Williams (1982) points out, it is these objects for which the detection of accretion-powered disks is more likely, since the collapse time for low-mass protostars is longer than for more massive ones. We note that some of the spectra are consistent with the emission expected from a disk of material accreting under the influence of gravity.

II. THE EMITTED SPECTRUM FROM ACCRETION DISKS IN PROTOSTELLAR SYSTEMS

The emitted spectrum from an accretion disk depends on a number of rather uncertain physical parameters. The opacity of the disk clearly depends on the dominant scattering mechanism but also depends on the amount of material present in the disk at a given radius. This in turn depends on the accretion rate and, therefore, the rate of mass transfer in the disk. Ultimately, the mass transfer rate depends upon the so-called "viscosity parameter," α, which may be a function of disk radius. Lin (1981) discusses these issues in some detail when he applies a modification of the Shakura-Sunyaev viscous disk model to the problem of disk accretion as a final stage in the formation of stars and planetary systems.

Given the uncertainty of the details of viscous disk models, it is appropriate to appeal to observations as much as possible to determine disk parameters. If we assume an optically thick disk, the specific intensity of the emission of each annulus of radius R is a Planck function of the form

$$B_\nu(R) = (2h/c^2)[\nu^3/e^{h\nu/kT(R)} - 1] \text{ ergs cm}^{-2}\text{ s}^{-1}\text{ Hz}^{-1}\text{ sr}^{-1}, \quad (1)$$

where h is Planck's constant, c is the velocity of light, k is the Boltzmann constant, and ν is the frequency at which the radiation is emitted. Allowing the Planck function to describe the emitted radiation ($B_\nu = I_\nu$) means that the specific flux incident on a detector will be

$$S_\nu = \int I_\nu d\Omega = (f/D^2)\int_{R_1}^{R_2} I_\nu(R)2\pi R dR \text{ ergs cm}^{-2}\text{ s}^{-1}\text{ Hz}^{-1}, \quad (2)$$

where D is the distance to the system, $f(= \cos\theta)$ is a projection factor which reduces the effective disk area when the disk is inclined with respect to the observer, and R_1 and R_2 are, respectively, the inner and outer radii of the disk for which the blackbody approximation holds.

The variation of disk temperature with radius is commonly assumed to be a power law, although as we noted above, the value of the exponent is uncertain. Therefore, we assume the disk temperature gradient of the form $T = T_0 r^{-p}$, where T_0 is the temperature of the disk at the radius of the protostar's surface (equal to R_0), $r = R/R_0$, and p is unspecified. Substituting this power-law dependence of temperature on disk radius, we find (Beall *et al.* 1984) that the specific flux is

$$S_\nu = 4\pi(f/D^2)(R_0^2 h/c^2)(kT_0/h)^{2/p}\nu^{(3-2/p)} \times \int_{x_1}^{x_2}(1/p)\{x^{[(2/p)-1]}/(e^x - 1)\}dx, \quad (3)$$

where we have made the substitution $x = h\nu/kT$, and where $x_1 = (h\nu/kT_0)(R_1/R_0)^p$ and $x_2 = (h\nu/kT_0)(R_2/R_0)^p$. In general, we do not assume that $R_1 = R_0$ in order to allow for the possibility that the blackbody approximation does not hold for the inner portion of the disk very near the protostar. The canonical value of $p = \frac{3}{4}$ (see, e.g., Lin 1981; Lynden-Bell 1969; Novikov and Thorne 1973; Shakhura and Sunyaev 1973) yields an emitted spectrum of $S_\nu \propto \nu^{1/3}$. Obviously, this spectrum extends only over a certain range of emitted frequencies. In fact, the maximum frequency for which the power law holds is the peak in the blackbody distribution for the inner edge of the disk. Thus, the maximum frequency for which the emitted spectrum is a blackbody is $\nu_{\max} \approx 2.8kT/h \approx 6 \times 10^{10} T_0 r_1^{-p}$, and the minimum frequency $\nu_{\min} \approx 6 \times 10^{10} T_0 r_2^{-p}$.

At frequencies greater than $\nu_{\max}$, the inner-edge emission from the disk will produce a Wien (exponential) tail. This tail may be observable if nuclear burning in the star has not yet begun. In the most general case, the spectrum may be a complex summation of the disk and stellar components at higher frequencies. At frequencies lower than $\nu_{\min}$, the disk spectrum will become that characteristic of the Rayleigh-Jeans limit ($S_\nu \approx \nu^2$) for a single-temperature blackbody.

From the point of view of observations, it is necessary to estimate the temperature, T_0, in order to determine the portion of the electromagnetic spectrum in which each of the three components of the disk spectrum (Rayleigh-Jeans, power law, or Wien) will fall. As we will see below, the power-law portion of the spectrum is likely to extend from the ultraviolet to the infrared in most cases.

It should be noted that the total luminosity of the disk can be used to estimate the luminosity of the disk annulus near R_0, because the gravitational energy released by the disk material as it moves from the outer edge of the disk down to R_0 is roughly equal to the energy released as the material moves the last incremental radius, dR, onto the protostar at R_0 (see, e.g., Pringle 1981).

Therefore, the temperature at the disk's inner edge, T_0, can be estimated by using the total luminosity of the disk (taken from infrared observations) to infer an accretion rate, $\dot{m} \approx L(R_0/\eta GM)$, where L is the disk luminosity estimated from infrared observations, R_0 is the radius of the protostar, η is the efficiency factor usually taken to be 0.1, G is the gravitational constant, and M is the mass of the protostar. This accretion rate is then used to determine an equilibrium temperature, $T_0 = (3GM\dot{m}/8\pi R_0^3\sigma)^{1/4}$, where σ is the Stefan-Boltzmann constant. Assuming that it radiates as a black body down to the radius R_0, the temperature which the disk reaches is T_0. We note that it is not necessary that the blackbody approximation hold in all cases down to R_0. In such a situation, the temperature, T_0, is not physical and must be regarded as simply a parameter which can be used to calculate the emitted spectrum. In this case, the disk emission would assume the character of a Wien tail at a frequency determined by the minimum radius at which the blackbody approximation does hold, as noted above.

The maximum extent of the disk can also be estimated by noting the frequency at which the disk emission changes from the power law to emission characteristic of a blackbody in the Rayleigh-Jeans limit ($S_\nu \propto \nu^2$ or $S_\lambda \propto \lambda^{-4}$).

Even if the absolute intensity of the disk spectrum cannot be readily determined, the slope of the power-law portion of the spectrum can yield an estimate of the temperature gradient of the disk. Observationally determined values of the accretion disk temperature gradient are important in a number of astrophysical problems, including the question of the formation of terrestrial planets, as noted earlier.

III. DISCUSSION

It is interesting to compare the foregoing theoretical discussion with observations taken at infrared frequencies. We will use the infrared data of Lada and Wilking (1984) from the ρ Oph cloud as representative of observations which are taken in dark clouds where star formation is likely to occur. This specific system was chosen because the ρ Oph cloud seems to contain low-luminosity, low-mass objects. Since low-mass protostars spend a longer time in the collapse phase, the detection of protostellar accretion disks for these sources becomes more likely.

An examination of the infrared data in the Lada and Wilking paper underscores the apparent complexity of the emitted spectra associated with the sources observed. In this discussion, references to figures should be taken to mean those in Lada and Wilking (1984). Our analysis is intended to be illustrative. The slopes mentioned are estimates and are not derived from least-square fits to these data. The reader should also note that Lada and Wilking plot λS_λ instead of S_λ. Thus, the slope on their plots with λ^{-3} becomes a λ^{-4} in a plot of S_λ. Their Figure 2 represents data taken from three sources, S1, GSS 23, and SR 3, in the ρ Oph dark cloud. As Lada and Wilking note, these spectra are consistent with single-temperature blackbody radiators suitably reddened due to interstellar extinction. Even with interstellar reddening not removed from the data, the spectrum at wavelengths greater than the K or L bands is a good approximation to the $\lambda^{-4}(\nu^2)$, which is the spectrum of a single-temperature blackbody in the Rayleigh-Jeans limit. GSS 23 and SR 3 show some apparent excess in the data taken from the N band.

The data from T Tauri stars in the ρ Oph cloud are presented in Figure 1 of Lada and Wilking's paper. These data show infrared excesses above a λ^{-4} spectrum in all the sources observed. The slope for SR 4 between the H and N infrared bands, and the slope for GSS 29 between the L and Q bands is $\beta \approx -1.8$, where $S_\lambda \propto \lambda^\beta$. This value of β corresponds to a value of $\alpha = -0.2$. In general, $\alpha + 2 = -\beta$ for an emitted spectrum which has the form of a power law (that is, $S_\nu \approx \nu^\alpha$ and $S_\lambda \approx \lambda^\beta$). Therefore, the power-law index of the disk temperature gradient, $p = 0.6$, where p is determined by $\alpha = 3 - 2/p$. The spectra of these two objects is thus inconsistent with a value of $p = \frac{3}{4}$, which is that customarily assumed for a viscous accretion disk. The power-law index is nearer the value of $p = 0.5$, which is consistent with a disk of material in radiative equilibrium with a central source, so the emission may represent this case. Most of the T Tauri sources in Lada and Wilking's Figure 1 share this approximate value of the power law for at least some portion of their spectra.

One may ask if there are any sources in the spectra presented by Lada and Wilking which have a canonical slope. If the emission does originate from an accretion disk with a temperature which follows a power law with the index, $p = \frac{3}{4}$, then the emitted spectrum within the limited range of frequencies mentioned above will be $\nu^{1/3}$ or $\lambda^{-7/3}$. On Lada and Wilking's scale, this would be represented by a slope of $-4/3$. In fact, source S2 between the N and Q bands in Figure 3, and, in Figure 4, sources WL20 and WL4 between K and N and L and Q, respectively, show such a spectrum.

It is reasonable to ask what value the central disk temperature would reach, given the working assumptions of this paper. If the energy source is due to accretion, and the energy loss is by blackbody radiation, the equilibrium temperature $T_0 = [(3L/8\pi\eta)(1/R_0^2\sigma)]^{1/4}$, where L is $\frac{1}{2}$ the observed luminosity, η is the efficiency factor in the conversion of gravitational energy, R_0 is the radius of the protostar, and σ is the Stegan-Boltzmann constant. Thus, $T_0 = 1.2 \times 10^1[(L^{1/4})/(R_0^{1/2})]$. For a source such as WL 20 with a luminosity of $\sim 0.2\ L_\odot$, and a protostar radius of order $3R_\odot$, $T_0 = 6 \times 10^3$. This value of temperature corresponds to an accretion rate of 4×10^{18} g s^{-1} onto a protostar of radius 2×10^{11} cm, where the disk is in thermal equilibrium with the energy input due to accretion.

This value of the central disk temperature, when combined with the radius, $R_0 = 2 \times 10^{11}$ cm, can be used to estimate the expected flux from an accretion disk in a particular band. For example, with the values as previously specified, and with $f = 1$, and $D = 160$ pc (the distance to the ρ Oph dark cloud) the estimated flux, $S_\nu = 2.2 \times 10^{-29}\nu^{1/3}$ ergs cm^{-2} s^{-1} Hz^{-1}. In the K band, WL 20 has $\lambda F_\lambda = 10^{-17}$ watts cm^{-2}, or 10^{-10} ergs cm^{-2} s^{-1}. Thus, the observed spectrum has a specific intensity of $S_\nu = 1 \times 10^{-24}$ ergs cm^{-2} s^{-1} Hz^{-1}. This agrees with the spectrum calculated above, since for the K band, $\nu = 1 \times 10^{14}$, and, therefore $S_\nu = 1 \times 10^{-24}$.

The maximum extent of disk can also be estimated from the infrared data, since the maximum radius for which the blackbody approximation holds will determine the frequency at which the spectrum changes from a power law to that of a single-temperature blackbody in the Rayleigh-Jeans limit. Since we have estimated $T_0 = 1 \times 10^4$, we note that the minimum frequency, $\nu_{\min} = 6 \times 10^8(R_2/R_0)^{-3/4}$, assuming that the temperature gradient, $p = \frac{3}{4}$. From the infrared data, it appears that $\nu_{\min} < 1 \times 10^{13}$ Hz. Therefore, $R_2/R_0 > 1.2 \times 10^2$. Since we have assumed that $R_0 = 2 \times 10^{11}$ for a protostar with $\sim 1\ M_\odot$, this implies that the maximum radius of the accretion disk is greater than 2.4×10^{13} cm. Since this is ~ 1 AU, the disk size is not beyond reasonable bounds for that which may be associated with a protostellar system.

The minimum mass of the disk can now be estimated if we assume that the density and height of the disk follow the Shakura and Sunyaev (1973) model. Taking the density,

$$\rho = 1.3 \times 10^{12}\mu^{9/8}\alpha^{-7/10}\dot{m}^{11/20}m^{5/8}R^{-15/8}\ \text{g cm}^{-3}\ ,$$

where μ is the molecular weight of the accreting material, α is the viscosity parameter, $\dot{m}$ is the accretion rate in units of $1.4\ M_\odot$, and R is the radial position within the disk; and the disk height,

$$h = 1.0 \times 10^{-3}\mu^{-3/8}a^{-1/10}\dot{m}^{3/20}m^{-3/8}R^{9/8}\ \text{cm}\ ,$$

the disk mass can be estimated as

$$M_{\text{disk}} = \int_{R_0}^{R_1} 2\pi R 2h\rho dR > 1.7 \times 10^{28}\ \text{g}\ .$$

For the model proposed by Lin (1981) a disk with a maximum radius greater than 2×10^{13} cm has a minimum mass $m_D > 9 \times 10^{28}$ g. If the nebula extends to a distance of 10^{14} cm (the orbit of Neptune), then (see, e.g., Lin 1981) the disk mass will be of order 10^{30} g. The disk mass we calculate is, therefore, a reasonable minimum value for the inner region of the protostellar system, since it excludes the protostar itself. If a turnover in the infrared spectrum for WL 20 can be found, it will be possible to place a definite value on the mass of the disk, given the working assumptions of a calculation such as this one.

The spectra of at least some of the ρ Oph sources are thus seen to be consistent with the model of a viscous, optically thick accretion disk. Given the complexity of the spectra, it is also possible that circumstellar shells of dust can contribute to the observed radiation (Stahler, Shu, and Tamm 1981). The actual, physical situation is unlikely to be fitted by any single model.

A final comment concerning possible confusion of detection of a disk with that of a star seems reasonable. The disk spectrum has a form which resembles a reddened blackbody with a higher temperature than that of the disk proper, since the disk spectrum flattens toward shorter wavelengths. Mass estimates of highly obscured objects where disks can be present are thus prone to this source of error. It has not escaped our notice that the discrepancy between infrared and ultraviolet estimates of the luminosity associated with stars in these regions may be relevant to this question.

The author tanks P. Schwartz, R. Silberberg, H. A. Smith, and K. S. Wood for helpful discussions. Helpful comments by an anonymous referee are also appreciated.

REFERENCES

Beall, J. H., *et al.* 1984, *Ap. J.*, **284**, 745.
Beichman, C. A., *et al.* 1984, *Ap. J (Letters)*, **278**, L45.
Boss, A. P. 1985, *Icarus*, **61**, 3.
Cameron, A. G. W. 1978, *Moon*, **18**, 5.
Elias, J. H. 1978, *Ap. J.*, **224**, 453.
Gehrz, R. D., Black, D. C., and Solomon, P. M. 1984, *Science*, **224**, 823.

Grasdalen, G. L., Strom, K., and Strom, S. E. 1973, *Ap. J.* (*Letters*), **184**, L53.
Hayashi, C., Nakazawa, K., and Adachi, J. 1977, *Pub. Astr. Soc. Japan*, **29**, 165.
Lada, C. J., and Wilking, B. A. 1984, *Ap. J.*, **287**, 610.
Lewis, J. S. 1974, *Science*, **186**, 440.
Lin, D. N. C. 1981, *Ap. J.*, **246**, 972.
Lin, D. N. C., and Papaloizou, J. 1980, *M.N.R.A.S.*, **191**, 37.
Lynden-Bell, D. 1969, *Nature*, **223**, 690.
Novikov, I. D., and Thorne, K. P. 1973, in *Black Holes*, ed. C. DeWitt and B. S. Dewitt (New York: Garden and Broach), p. 344.
Pringle, J. E. 1981, *Ann. Rev. Astr. Ap.*, **19**, 137.
Safronov, V. S. 1972, *On the Origin of the Solar System* (Paris: CNRS), p. 89.
Shakura, N. I., and Sunyaev, R. A. 1973, *Astr. Ap.*, **24**, 337.
Smak, J. 1985, in *Proc. IAS/CNR Workshop on Multifrequency Behavior of Galactic Accreting Sources* (*Frascati*), ed. F. Giovanelli (Vulcano: Italian Astr. Soc.), p. 3.
Stahler, S. W., Shu, F. H., and Taam, R. E. 1980, *Ap. J.*, **241**, 637.
———. 1981, *Ap. J.*, **248**, 727.
Verbunt, F. 1982, *Space Sci. Rev.*, **32**, 379.
Vrba, F. J., Strom, K. M., Strom, S. E., and Grasdalen, G. L. 1975, *Ap. J.*, **197**, 77.
Welch, W. J., Vogel, S. N., Plambeck, R. L., Wright, M. C. H., Bieging, J. H. 1985, *Science*, **228**, 1389.
Wetherill, G. W. 1978, in *Protostars and Planets*, ed. T. Gehrels (Tucson: University of Arizona Press), p. 565.
Wynn-Williams, C. G. 1982, *Ann. Rev. Astr. Ap.*, **257**, 171.

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